Quiz 7: 15.1-15.3

Show all work clearly. Any time a substitution is made in an integral, or integration by parts is used, you must state what substitution you are using.

1) Evaluate the double integral $\int_1^3\int_1^5 \frac{\ln y}{xy}dydx$ (5 points)

$$\int_1^3\int_1^5 \frac{\ln y}{xy}dydx = \int_1^3 \frac{1}{x}\int_1^5 \frac{\ln y}{y}\,dy\,dx$$

$u = \ln y$

$du = \frac{1}{y}dy$

$$\int_1^5 \frac{\ln y}{y}\,dy = \int_0^{\ln 5} u\,du = \frac{1}{2}u^2\Big]_0^{\ln 5}$$

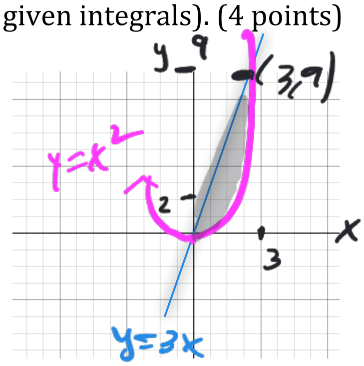
$$= \int_1^3 \frac{1}{x}\,\frac{1}{2}(\ln 5)^2\,dx$$

$$= \frac{1}{2}(\ln 5)^2 \int_1^3 \frac{1}{x}\,dx$$

$$= \frac{1}{2}(\ln 5)^2 \ln|x|\Big]_1^3 = \frac{1}{2}(\ln 5)^2 \ln 3$$

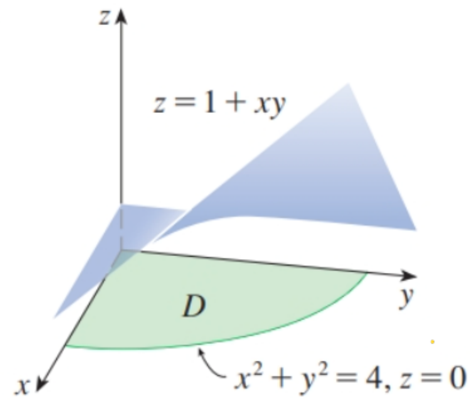
(2) Set up iterated integrals in both orders for $\iint_D f(x,y)\,dA$, where D is the region bound by $y = x^2;\ y = 3x$. (write your limits on the given integrals). (4 points)

$$\int_0^3 \int_{x^2}^{3x} f(x,y)\,dy\,dx$$



$$\int_0^9 \int_{\frac{1}{3}y}^{\sqrt{y}} f(x,y)\,dx\,dy$$

(3) Find the volume of the solid under the surface $f(x,y) = 1 + xy$ and above the region in the D in the xy plane shown, using the following instructions. (10 points)



(a) Set up only: a double integral in the order dydx

$$\int_0^2 \int_0^{\sqrt{4-x^2}} (1+xy)\,dy\,dx$$

(b) Set up only a double integral in the order dxdy

$$\int_0^2 \int_0^{\sqrt{4-y^2}} (1+xy)\,dx\,dy$$

(c) Set up only in polar coordinates.

$$\int_0^{\pi/2} \int_0^2 (1+r^2\cos\theta\sin\theta)\, r\, dr\, d\theta$$

(d) Evaluate one of the above

$$\int_0^{\pi/2} \int_0^2 (r + r^3(\cos\theta\sin\theta))\, dr\, d\theta$$

$$\int_0^{\pi/2} \frac{1}{2}r^2 + \frac{1}{4}r^4 \cos\theta\sin\theta \Big]_0^2 \, d\theta$$

$$\int_0^{\pi/2} (2 + 4\cos\theta\sin\theta)\, d\theta$$

$$\int_0^{\pi/2} 2\, d\theta + \int_0^{\pi/2} 4\cos\theta\sin\theta\, d\theta$$

$$2 \cdot \frac{\pi}{2} + 2\sin^2\theta \Big]_0^{\pi/2}$$

$$\boxed{\pi + 2}$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$\int \cos\theta\sin\theta\, d\theta$

$= \int u\,du = \frac{1}{2}u^2 + C$

$= \frac{1}{2}\sin^2\theta + C$